

$b$  = lateral displacement from CofM

$$b = \frac{z_1 z_2 k e^2}{2.E} \cot \frac{\theta}{2}$$

let  $s = \frac{z_1 z_2 k e^2}{2.E}$

for an inertial frame with  $D_1$  stationary:

$$\text{relative energy loss } K = \sin^2 \frac{\theta}{2}$$

$$\text{so } \sin \frac{\theta}{2} = \sqrt{K}, \quad \cos \frac{\theta}{2} = \sqrt{1-K}$$

$$b = s \frac{\sqrt{1-K}}{\sqrt{K}} \Rightarrow b^2 = s^2 \frac{1-K}{K} \Rightarrow b^2 + s^2 = \frac{s^2}{K} \Rightarrow K = \frac{s^2}{b^2 + s^2}$$

Now sum relative energy losses for  $b$  to radius  $R$   
and for all angles about axis parallel to projectile  
motion through CofM.

$$\begin{aligned} \text{sum of energy losses} &= \int_0^{2\pi} \int_0^R K \cdot db \cdot b \cdot d\theta = 2\pi s^2 \int_0^R \frac{b}{b^2 + s^2} db \\ &= \pi s^2 \left[ \ln |R^2 + s^2| - \ln |s^2| \right] \\ &= \pi s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right| \end{aligned}$$

this sum is with respect to a cross-section of <sup>beam of</sup>  $D_2$  to radius  $R$   
such that it is over an area of  $\pi R^2$

$$\text{so average energy loss } \bar{K} = \frac{s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right|}{R^2}$$

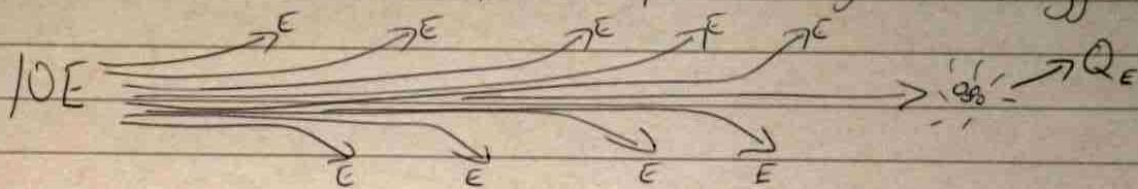
therefore the beam cross-section can be written:

$$\pi R^2 = \frac{\pi \cdot s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right|}{\bar{K}}$$

①



Consider a 'beam-type' fusion process such that (by example) a target particle fuses one time out of ten; and the particles begin with energy  $E$



Here, an input of  $10E$  loses  $9E$  before releasing  $Q_E$  of energy

so to release as much energy as put in:

$$\frac{P_f}{P_s} \approx \frac{\text{probability of fusing}}{\text{probability of scattering}} = \frac{1}{9} > \frac{E}{Q_E}$$

we define a scattering cross-section,  $\sigma_c$ , and compare it with a fusion cross-section,  $\sigma_f$ , such that:

$$P_f = \frac{\sigma_f}{\sigma_f + \sigma_c} \quad \text{and} \quad P_s = \frac{\sigma_c}{\sigma_f + \sigma_c}$$

so that for over-unity energy out:

$$\frac{\sigma_f}{\sigma_c} > \frac{E}{Q_E}$$

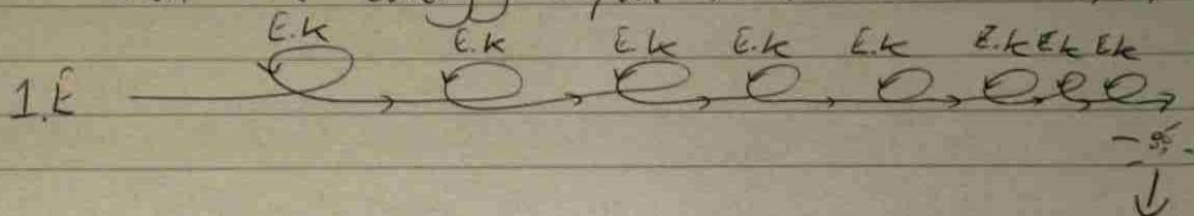
Where this  $\sigma_c \gg \sigma_f$  we can say the cross-section of ① will contain mostly particles that will scatter, i.e.  $\sigma_c = \pi R^2$

so for over-unity energy out:

$$\textcircled{2} \quad \frac{\sigma_f \cdot \bar{k}}{\pi \cdot S^2 \cdot \ln \left| \frac{R^2 + S^2}{S^2} \right|} > \frac{E}{Q_E} \Rightarrow \sigma_f > \frac{\pi \cdot E \cdot S^2 \cdot \ln \left| \frac{R^2 + S^2}{S^2} \right|}{\bar{k} \cdot Q_E}$$

THIS CAN BE PROVED FALSE AS  $\bar{k}$  WILL TEND TO ZERO FOR VERY LARGE  $R$  SO RHS BECOMES INFINITELY LARGE.

Consider an alternative fusion process in which scattered particles are recycled, such that the energy input is the scattered loss;  $k$ ;



there is an input of  $E + 9kE$  before the release of energy,  $Q_E$

[ Here, we recognize that in reality there will be several thousand scattering events for each fusion such that the result would tend to  $9k$ . ]

So to release as much energy as put in

$$\frac{P_f}{P_s} = \frac{1}{9} > \frac{E \cdot k}{Q_E}$$

and (2) can be re-written:

$$\frac{\sigma_f \cdot k}{\pi \cdot s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right|} > \frac{E \cdot k}{Q_E} \Rightarrow \sigma_f > \frac{\pi \cdot E \cdot s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right|}{Q_E}$$

note that this form neglects the input 'E' for the initial energy of the recycled particle. This can be re-inserted into the equation by treating the 'net output energy';  $Q_E - E$

So for over-unity energy:

$$(3) \quad \sigma_f > \frac{\pi \cdot E \cdot s^2 \cdot \ln \left| \frac{R^2 + s^2}{s^2} \right|}{Q_E - E}$$