



Magnetic lenses for neutron beams

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Abstract

Purpose – To provide information on magnetic lens design principles for neutron beam focussing and to characterize the efficiency of these devices.

Design/methodology/approach – Magnetostatic and neutron optical computations are presented for permanent magnet and electromagnetic hexapole lenses for neutron beam focusing. The numerical results are verified by magnetic and neutron beam measurements.

Findings – Neutron lenses built from brick-shaped permanent magnets approximating the six-pole Halbach structure yield relatively high field constant for a relatively small useful diameter. Electromagnets are weaker, but allow field constant adjustment.

Research limitations/implications – The main limitation of both presented solutions resides in the difficulty of obtaining high magnetic flux densities. Superconducting coils in hexapole configuration can overcome this limitation.

Practical implications – The magnetic lenses can be used for beam focusing in scattering neutron instruments. The focused beam is also polarized.

Originality/value – Analytical formulae are supplied for the neutron path calculation in a six-pole magnetic field. An electromagnet design with permanent magnet flux enforcement is provided.

Keywords Magnetic devices, Numerical analysis, Optical instruments

Paper type Research paper

1. Introduction

Six-pole magnetic fields yield flux density gradient proportional to radius and act as lenses upon neutrons flying along the field. Neutrons with spins oriented parallel to the field are defocused while those having opposite orientation are focused. The hexapole magnets act as lenses upon neutron beams (Farago, 1964).

2. Movement of neutrons in magnetic field

The neutron is endowed with angular momentum L , constant in magnitude and magnetic spin s , connected to the former by

$$s = \gamma L, \quad (1)$$

where γ is the giromagnetic ratio of the neutron. The neutron evolves according to the equations of motion of a classical magnetic moment (Mezei, 1972).

The force and torque exerted on the neutron in magnetic field can be expressed:

$$F = (s \cdot \nabla)B; \quad T = s \times B \quad (2)$$

and the equations governing its motion are:

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$$\mathbf{F} = m \frac{d^2 \mathbf{r}}{dt^2}; \quad \mathbf{T} = \frac{d\mathbf{L}}{dt} = \boldsymbol{\omega} \times \mathbf{L}, \quad (3)$$

with $\boldsymbol{\omega}$ the angular velocity of Larmor precession.

According to equation (2), inhomogeneous magnetic field is required to control the trajectory of a neutron (the shape of a neutron beam). A very long, ideal $2n$ -pole magnet produces a scalar potential expressed in cylindrical coordinates:

$$\Psi = Cr^n \cos n\varphi, \quad (4)$$

which corresponds to a flux density:

$$\mathbf{B} = \mu_0 C n r^{n-1} (-\cos n\varphi \mathbf{e}_r + \sin n\varphi \mathbf{e}_\varphi), \quad (5)$$

leading to the force exerted upon the neutron:

$$\mathbf{F} = s_r \frac{\partial \mathbf{B}}{\partial r} + s_\varphi \frac{1}{r} \frac{\partial \mathbf{B}}{\partial \varphi} \quad (6)$$

It can be shown that the six-pole magnets ($n = 3$) can be used as lenses for neutron beam focusing (Farago, 1964).

Experiment shows that the neutrons are oriented adiabatically by the stray field of the magnet, either parallel or anti-parallel to the magnetic field. A solenoid with axial field can also be used to ensure the fulfillment of this condition. In such case the expression of the force becomes

$$\mathbf{F} = -s \nabla |\mathbf{B}| = -2sC \mathbf{r}. \quad (7)$$

This equation leads to the equation of motion:

$$\frac{d^2 \mathbf{r}}{dt^2} + \omega^2 \mathbf{r} = 0; \quad \omega^2 = \frac{2sC}{m}, \quad (8)$$

with initial conditions:

$$\mathbf{r}(0) = \mathbf{r}_0; \quad \frac{d\mathbf{r}}{dt}(0) = \mathbf{v}_0. \quad (9)$$

In Cartesian coordinates this equation allows separation of variables and the analytical solution can be obtained:

$$\begin{cases} x = x_0 \cos(\omega t) + \frac{v_{x0}}{\omega} \sin(\omega t) \\ y = y_0 \cos(\omega t) + \frac{v_{y0}}{\omega} \sin(\omega t) \end{cases}, \quad (10)$$

where (x_0, y_0) are the coordinates of the neutron with respect to the magnet axis at the moment it enters the field and (v_{x0}, v_{y0}) are the components of the transversal velocity at the same instant. The axial velocity of the neutron (v_z) in a beam is much higher than the transverse ones and can be considered to be constant.

The transversal components of the neutron velocity during the flight in the six-pole field are:

$$\begin{cases} v_x = v_{x0} \cos(\omega t) - \omega x_0 \sin(\omega t) \\ v_y = v_{y0} \cos(\omega t) - \omega y_0 \sin(\omega t) \end{cases} \quad (11)$$

These equations allow the computation of the position and velocity of the neutron at any moment. They can also serve as accuracy assessment of numerical solvers based on discretization in time domain applied in the case of more complex field configurations.

Neutrons with zero angular momentum with respect to the lens axis ($\mathbf{v}_0 \times \mathbf{r}_0 = 0$) are focused into one point, the focal point of the lens. The neutrons which possess non-zero initial angular momentum travel along a path with elliptical projection on the transversal plane, the angular momentum being conserved throughout the motion (Figure 1).

The initial conditions (beam size and divergence) are defined by the neutron guides and collimators upstream of the lens position. The paths of neutrons having an orbital angular momentum, projected onto a transverse plane in a six-pole field of 40,000 T/m², lens length 1,000 mm, lens radius 5 mm, neutron axial velocity 40 m/s (100 Å – very slow, so-called ultra cold neutron), respectively, 659 m/s (6 Å – so-called cold neutron) are plotted in Figure 1; the projection of the upstream and downstream flight sections are straight lines, the curved sections show the path inside the lens field.

Real magnet assemblies generally do not produce clean six-pole fields. The most disturbing inaccuracy is the presence of a bipolar component, which leads to double minima in the flux density magnitude distribution. In the vicinity of the beam axis the six-pole field magnitude is close to zero, therefore even a relatively weak dipole field can produce a significant effect. The flux density in this case is expressed as:

$$\begin{aligned} \mathbf{B} = & [-C_1 \cos(\varphi - \varphi_1) - C_3 r^2 \cos 3(\varphi - \varphi_3)] \mathbf{e}_r + [C_1 \sin(\varphi - \varphi_1) \\ & + C_3 r^2 \sin 3(\varphi - \varphi_3)] \mathbf{e}_\varphi \end{aligned} \quad (12)$$

and the contour plot is shown in Figure 2, where parabolic scale has been applied for better resolution in the central region (Figure 2).

In such a field even a neutron with initial velocity parallel to the lens axis travels along a complex path (Figure 3). In Figure 4 the computation results can be seen for the neutron velocities as in Figure 1. In these computations a dipole component of 50 mT has been considered, with the same orientation along the whole magnet length.

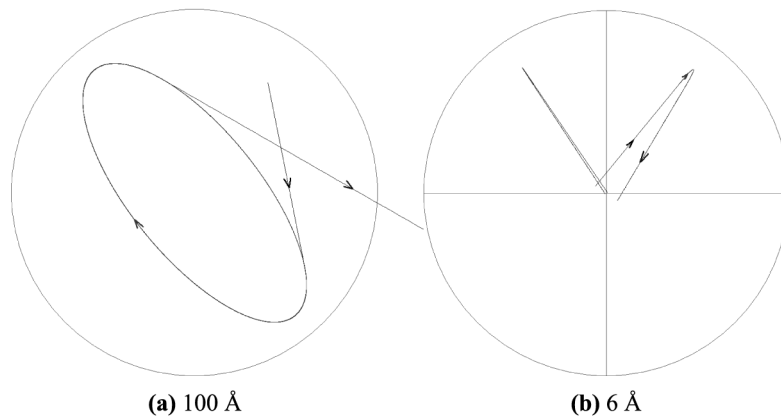


Figure 1.
Paths of neutrons with
orbital angular
momentum, projected onto
a transverse plane in
a six-pole field

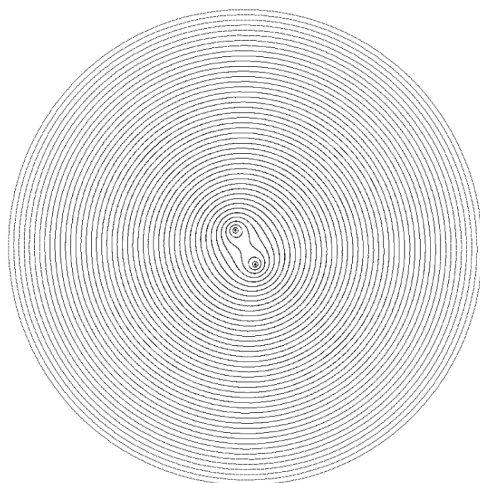


Figure 2.
The contour plot of flux
density distribution in a
hexapole field with bipolar
component

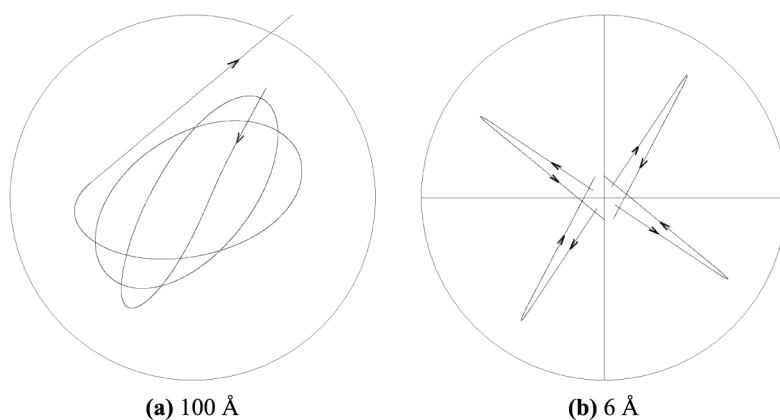


Figure 3.
Paths of neutrons with
zero initial orbital angular
momentum, projected onto
a transverse plane in
six-pole field with bipolar
component

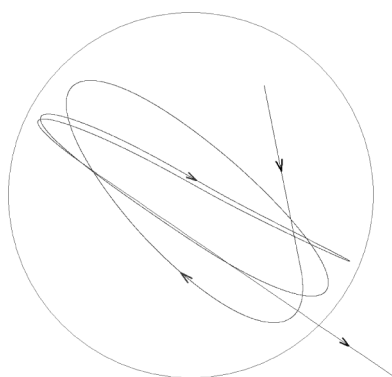


Figure 4.
Path of a 100 Å neutron
with orbital angular
momentum, projected onto
a transverse plane in a
six-pole field with bipolar
component

The computations have been repeated for a lens consisting of six segments, each segment having different orientation of the dipole component ($-\pi/3, 0, \pi/3, 2\pi/3, \pi$, and $-2\pi/3$ in the order of the neutron flight direction). The results are plotted in Figure 5.

Another source of inaccuracy resides in the fact that generally a neutron beam is not monochromatic. The existing monochromators yield a selected central wavelength (λ) and a distribution around it, characterized by $\Delta\lambda/\lambda$. The focal length of a magnetic lens in focus-to-focus configuration (defined as the distance from the magnet median plane to the plane where a divergent beam originating from a point in the upstream focal plane is focused) can be written

$$F = \frac{l}{2} + \frac{v_n}{\omega \tan \frac{\omega l}{2v_n}}, \quad (13)$$

where v_n is the velocity of the neutron, related to its wavelength by

$$v_n = \frac{3956}{\lambda[\text{\AA}]} \text{ m/s} \quad (14)$$

The image of a point-like source in the upstream focal plane will be a spot in the downstream focal plane, due to the chromatic aberration (neutrons with different velocities are focused in different planes).

The largest accepted radius at the magnet entrance and the largest incoming beam divergence in the point-to-point configuration are:

$$r_{0\max} = r_m \cos \frac{\omega l}{2v_n}; \quad \tan \alpha_{\max} = \frac{\omega r_m}{v_n} \sin \frac{\omega l}{2v_n}, \quad (15)$$

where r_m is the radius of the lens aperture. The radius of the focal spot in the case of a beam containing neutrons with velocities ($v_n \pm \Delta v$) can be estimated to be:

$$r_f = \frac{r_0}{\cos \frac{\omega l}{2v_n}} \left[\left(1 + \frac{\Delta v}{v_n} \right) \frac{\sin \frac{\omega l \Delta v}{v_n^2}}{\tan \frac{\omega l}{2v_n}} - \cos \frac{\omega l \Delta v}{v_n^2} \right], \quad (16)$$

which is a measure of the chromatic aberration.

Simulation of neutron beam focusing with a magnetic lens under the following conditions: beam divergence: 15° , wavelength: $6 \text{ \AA}$ ($v_z = 660 \text{ m/s}$), magnet length:

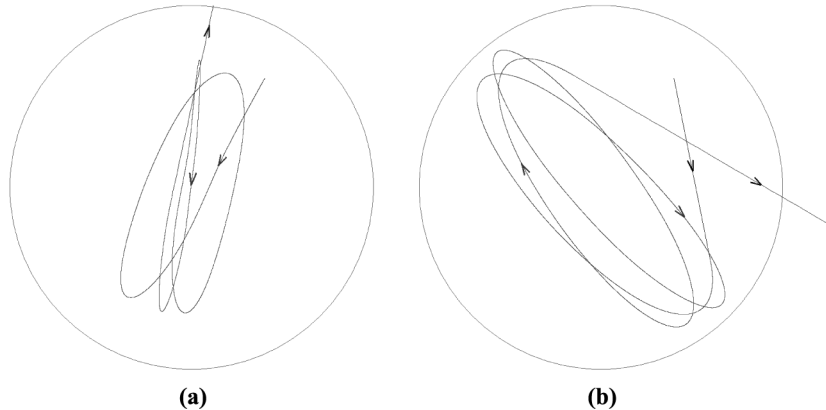


Figure 5.
100 Å neutron transverse
flight paths in six-pole
field with bipolar
component having
different orientations in
six lens segments

900 mm, hexapole field constant: 40,000 T/m², dipole field: 20 mT (same orientation in each magnet segment) leads to the results shown in Figure 6. When the dipole component is oriented differently in different magnet segments the error is averaged out and an isotropic beam profile results (Figure 7).

3. Lens design

Electromagnetic design, magnetic field map, computed and experimental neutron optical characteristics of permanent magnet and electromagnetic hexapole neutron lenses are presented.

The aim is to obtain a magnetic field as strong and as close to the ideal six-pole configuration as possible.

Six-poled fields can be realized using permanent magnet pieces with magnetization periodically rotated in space. When the magnetic material fills the available space and the magnetization rotates smoothly from point to point (Halbach structure (Halbach, 1980)) the strongest field is obtained (Schiemenz *et al.* 1991). A design

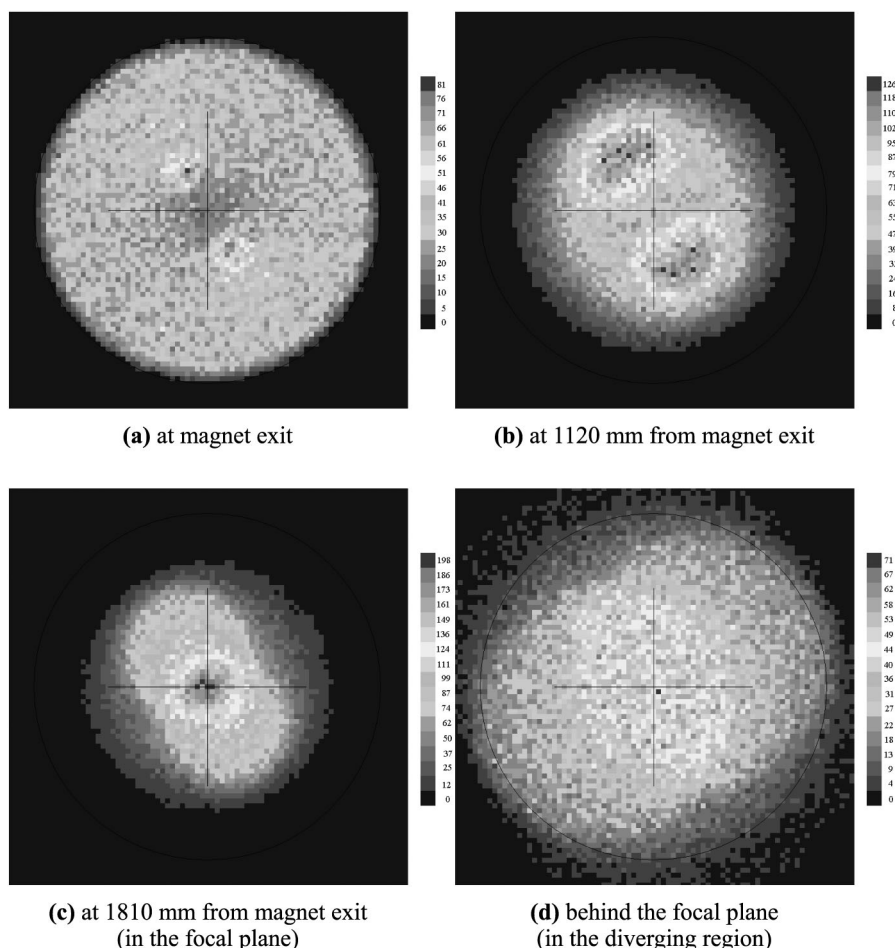


Figure 6.
Simulated beam profiles
focused with a lens having
systematic bipolar
component

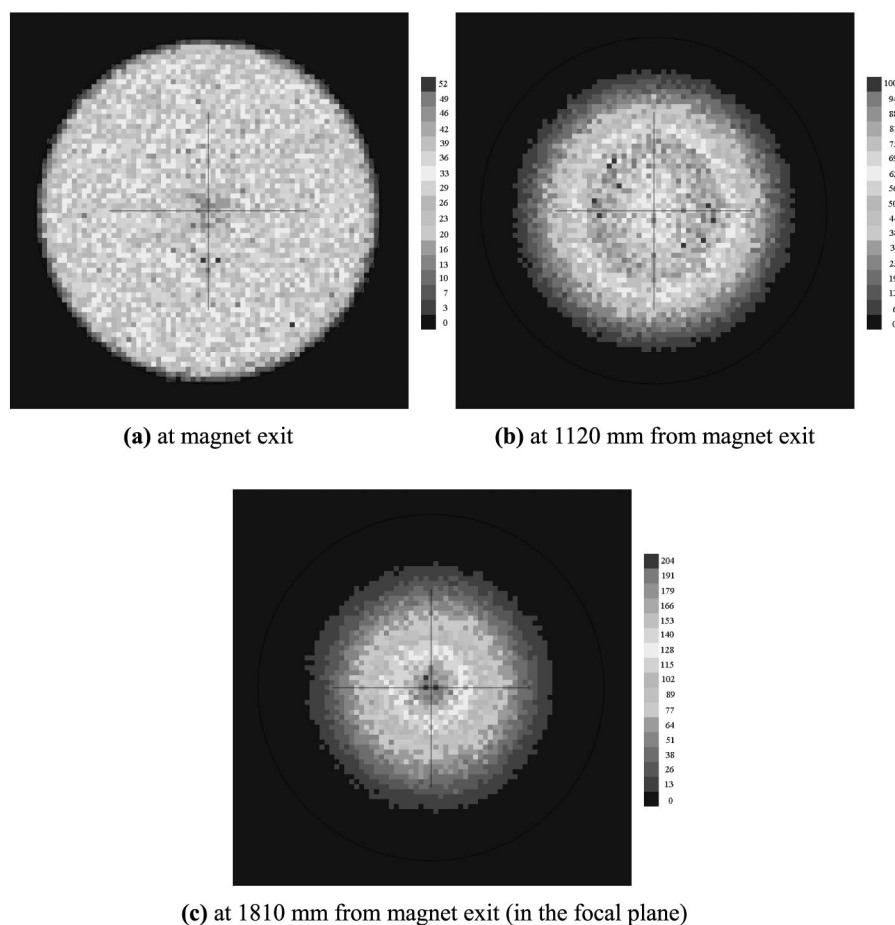


Figure 7.
Simulated beam profiles
focused with a lens having
randomly oriented bipolar
component

approximating the Halbach structure using brick-shaped NdFeB permanent magnets has been realized, achieving two thirds of the ideal field strength (Plate 1).

For the electromagnet design high coercivity permanent magnets are used to hinder the flux leakage between neighbouring poles outside the central airgap. Higher hexapole field constant (higher efficiency) is obtained this way, maintaining the advantage of adjustable parameters. The magnetic field is computed by means of the integral equation method. Clear six-pole field configuration – reduced dipolar component – is ensured by individual adjustment of the pole coil supply currents.

The cross-section of the electromagnet is shown in Figure 8 with iron yoke, coils, water cooling channels and flux enforcing permanent magnets.

Figure 9 shows the measured flux density between the poles of the built electromagnet in the absence of the permanent magnets, the useful flux density at the margin of the airgap with and without the permanent magnets in place. The airgap diameter is 16 mm. Achieved hexapole constant: 15,000 T/m². The computed magnetization vectors in the iron poles without permanent magnets are shown in

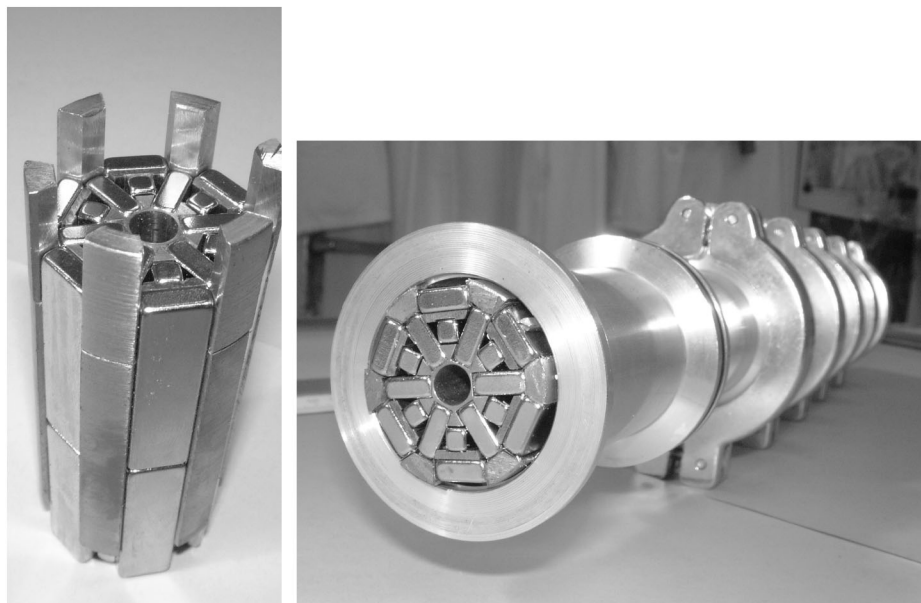


Plate 1.
Permanent magnet
hexapole segment and
assembled lens

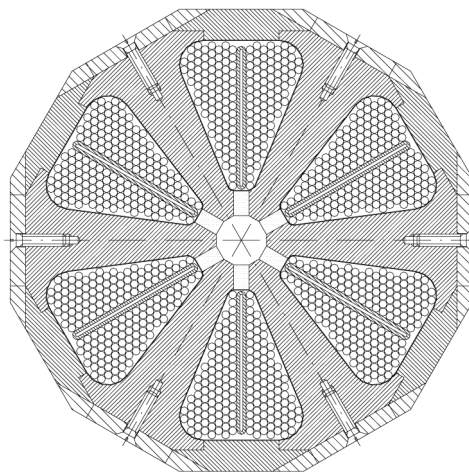


Figure 8.
Cross-section of the
flux-enforced
electromagnet

Figure 9. Flux leakage between poles and saturation of pole tips limit the field strength in the airgap. The role of the permanent magnets is to diminish this effect (Figure 10).

4. Experimental results

The magnetic field is mapped with a small-size Hall probe mounted in an appropriate holder (Füzi *et al.*, 2002).

The neutron optical characteristics of the constructed permanent magnet hexapole lenses have been verified by experiments on a cold neutron beam with a 2D position sensitive detector in time-of-flight regime (Füzi *et al.*, 2004).

The efficiency of each lens is evaluated by the focal length, size and sharpness of the focal spot and the number of focussed neutrons with respect to the incoming ones at several wavelengths. Figures 11 and 12 show the measured results, yielding the beam cross-section in the converging (*a* and *b*),

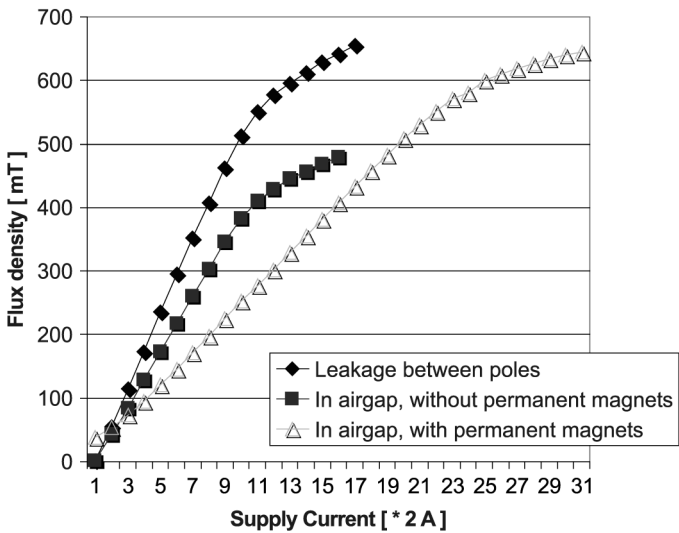


Figure 9.
Magnetic flux density
measured in the hexapole
electromagnet airgap

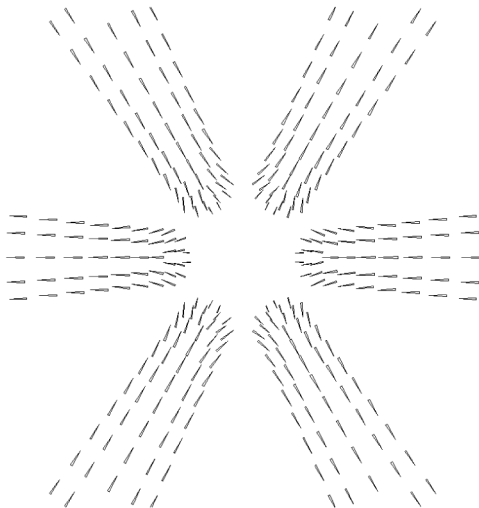


Figure 10.
Magnetization vectors of
the surface elements in the
central region

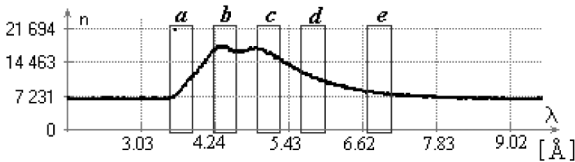


Figure 11.
Wavelength slices for
plotted images

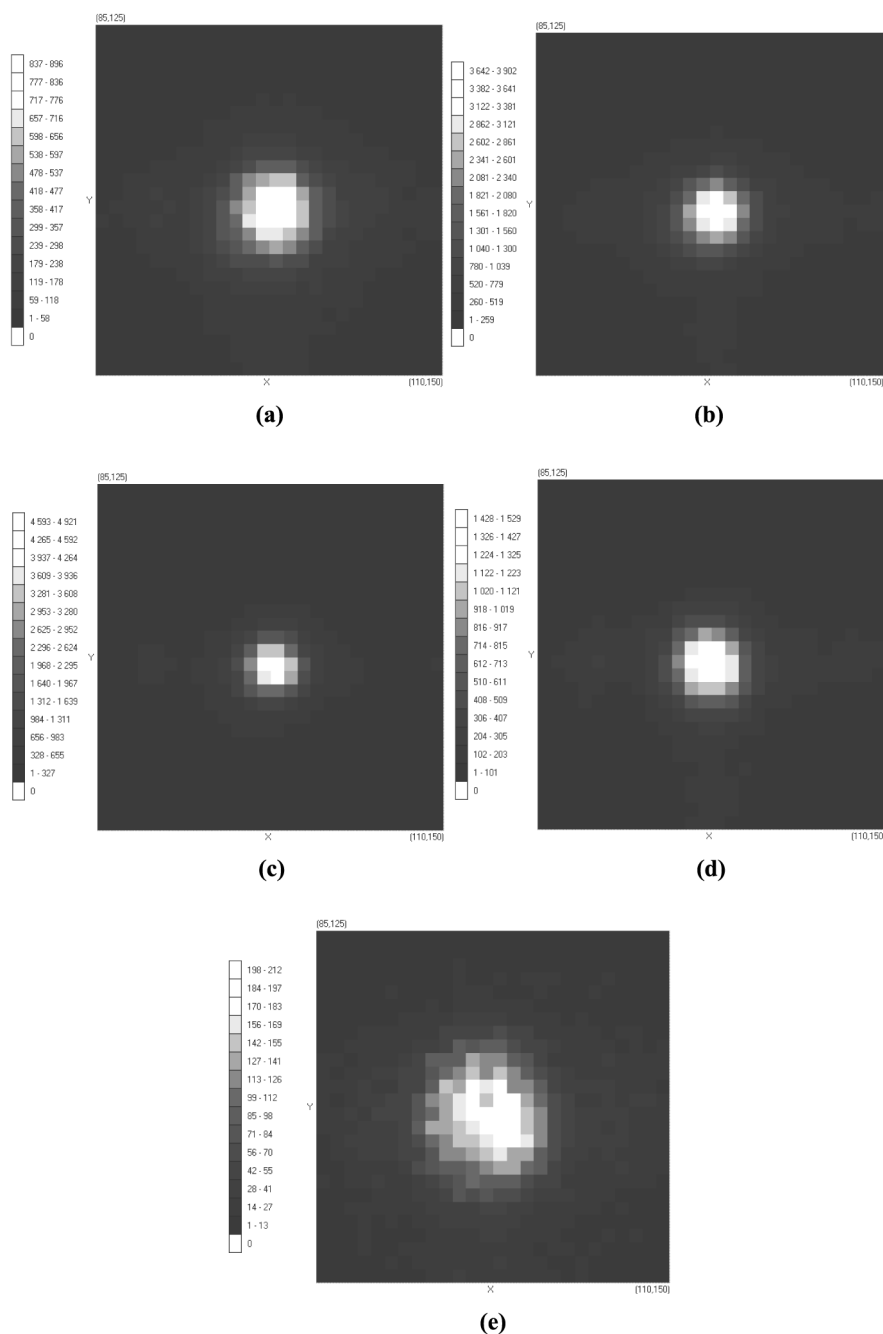


Figure 12.
Beam profile at various
wavelengths

focussed (*c*) and diverging (*d* and *e*) sections for the given wavelength and detector position.

The focal distance of the lenses depend on the magnet constant and the neutron wavelength (velocity) since the equations of motion in transversal direction are independent of the axial velocity, which determines the time spent by the neutron in the magnet field. Both the magnetic and neutron beam measurements lead to the result that the 450 mm long, 7 mm diameter airgap permanent magnet lenses have a hexapole constant of 47,000 T/m².

5. Conclusion

Two types of magnetic neutron focusing devices have been designed and constructed at the Budapest Neutron Centre and their characteristics experimentally verified.

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